The influence of finite and infinite wall cavities on the sound insulation of double-leaf walls

Cambridge, Jason; Davy, John; Pearce, John

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2 The influence of finite and infinite wall cavities on the sound insulation of double-leaf walls
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4

5 Jason E. Cambridge¹

6 University of Canterbury, Mechanical Engineering, Private Bag 4800, Christchurch 8140,
7 New Zealand
8
9

10 John L. Davy²

11 School of Science, Royal Melbourne Institute of Technology (RMIT) University,
12 GPO Box 2476 Melbourne, Victoria 3001, Australia
13
14

15 John Pearce

16 University of Canterbury, Mechanical Engineering, Private Bag 4800, Christchurch 8140,
17 New Zealand
18

19 Running title: Insulation of finite and infinite walls
20
21

¹ Author to whom correspondence should be addressed. Electronic mail:
JasonCambridge@hotmail.com

² Current address: CSIRO Infrastructure Technologies, Private Bag 10 Clayton South, Victoria
3169, Australia.

Theories used to predict the sound insulation of double-leaf cavity wall systems are usually based on the assumption that the wall is of an infinite extent. To account for the effect of the finite extent of the wall, a limiting the angle of incidence, a finite radiation efficiency model or the spatial windowing method are used in order to obtain realistic predictions. However, the effects of the finite extent of the cavity are often not included. This paper presents an extension of a finite two-dimensional cavity theory to include limp panels on each side of the cavity. It is shown that the oblique incidence mass-air-mass resonance can only occur for certain frequencies and certain angles of incidence. This is the reason why the infinite extent theories under predict the sound insulation. The results of the predicted sound insulation agree with measurements when the wall cavity is empty. To obtain agreement when the cavity is full of a porous sound absorbing material, a flow resistivity of about one fifth of the measured value has to be used. Use of the actual flow resistivity gives sound insulation values that are 10 dB too high.

35 I. INTRODUCTION

36 The prediction and understanding of the sound insulation of double-leaf cavity wall
37 systems has dramatically improved since the development of the early classical prediction
38 models by Beranek and Work (1949) and London (1950). However, despite these improvements,
39 a detailed survey by Hongisto (2006) of more than twenty well-known prediction models
40 concluded that there is a high degree of variability in the results produced from these models and
41 no single model was capable of predicting the sound insulation of the entire spread of
42 commercially available walls. Thus there is still a need to develop better sound insulation
43 prediction models for double-leaf cavity walls.

44 Many of the theories used to predict the sound insulation of a wall assume that the wall is
45 of infinite extent. A typical example is the transfer matrix method which is well described by
46 Allard and Atalla (2009). To account for the effects of the finite size of the wall on the radiation
47 efficiency, these theories sometimes use a finite size radiation efficiency [see Appendix 12.A of
48 (Allard and Atalla, 2009) and Section II.H of (Brunskog, 2012)]. A variant of this approach is the
49 spatial windowing technique [see (Villot *et al.*, 2001), (Villot and Guigou-Carter, 2005) and
50 (Vigran, 2009)].

51 For single leaf walls, Sewell (1970) showed that below the critical frequency, the finite
52 size radiation efficiency can be included by using a limiting angle of integration which depends
53 on the size of the wall compared to the wave length of sound.

54 For empty double-leaf cavity wall systems limiting the angle of integration does not
55 produce reasonable results. Consequently, an artificially high rate of sound attenuation is often
56 applied. For instance, while utilizing the transfer matrix method, Vigran (2009) sets “the power

attenuation coefficient equal to 0.2 m^{-1} independent of frequency” and uses a cavity depth which is slightly greater than the actual cavity depth. Davy (2010) also utilized an artificially high value of effective cavity sound absorption and introduced regression equations for the effective sound absorption of an empty wall cavity as a function of cavity depth. London (1950) used a panel resistance which varied with angle of incidence to overcome the problem of empty cavities.

The research described in this paper presents an analytic two-dimensional model for predicting the sound insulation of double-leaf cavity walls below their critical frequency. The model enables a better understanding of the role of the finite cavity on the sound insulation of double-leaf cavity walls and provides an explanation for the reason why an artificially high rate of attenuation is needed within the theories which assume that the walls are of infinite extent.

Gösele and Gösele’s (1977) model is extended by assuming that the two major air cavity boundaries are limp panels with possibly different specified mass per unit area. This assumption means that the solution is only valid to approximately half the critical frequencies of the panels. The panels were assumed to be loaded on their sides opposite the air cavity with the infinite or finite size panel radiation resistance when the panel was radiating into an infinite half space. The radiation reactance was ignored.

Because the existing theories of panel radiation resistance do not cover panel waves which are being attenuated with distance, the panel radiation resistance was calculated from first principles using a two-dimensional model. The results obtained are in good agreement with three dimensional models except at very low frequencies where the two-dimensional radiation efficiencies are higher than the three dimensional radiation efficiencies Ljunggren (1991), Davy (2009a).

Prasetiyo (Prasetiyo, 2012; Prasetiyo and Thompson, 2012) has used the coupled Waveguide Finite Element-Wave domain Boundary Element method to obtain results similar to this paper for the angular dependence of the sound insulation of double-leaf cavity walls of finite extent without sound insulation in their air cavities. One difference from this paper is that Prasetiyo does not find the normal incidence mass-air-mass resonant frequency. This result of Prasetiyo appears to be at odds with the experimental measurements of Brutel-Vuilmet *et al.* (2006). Prasetiyo, Brutel-Vuilmet *et al.* (2005) and this paper all reach the conclusion that the resonance frequencies of cavity walls of finite extent do not vary with the angle of incidence. This observation explains why London's (1950) model underestimates the sound insulation of cavity walls unless an artificial angular dependent panel resistance is introduced. It also explains why the other infinite extent models have to introduce artificially high values for cavity absorption, damping or attenuation.

II. GOVERNING EQUATIONS

In this paper, the time dependency is assumed to $e^{j\omega t}$ and is omitted from the equations. Consider the sketch of a double-leaf wall system shown in Figure 1. Assuming that each wall panel acts as a limp mass; the specific acoustic impedance (Z_i) of each wall panel can be found by taking into account the fluid loading effect of the surrounding air on each panel by considering its radiation efficiency (σ_i) and the characteristic impedance ($\rho_0 c$) of the air such that

$$Z_i = j\omega m_i + \sigma_i \rho_0 c, \quad (1)$$

where the mass per unit area of each panel is m_i .

Figure 1 Co-ordinate system, incident and bending wave number used within the proposed model (color online)

The specific acoustic impedance (Z_i) of each wall panel is defined as the ratio of the sound pressure acting on it to its particle velocity (v_i). Assuming that a plane harmonic wave is incident on panel one, Gösele and Gösele (1977) showed that once the depth of the wall cavity is small compared to the wavelength of the incident sound waves, a description of the air waves excited in the cavity parallel to the x-direction is sufficient for the required analysis. Consequently, the incident sound pressure (p_i) can be modelled as,

$$p_i(x) = \hat{p}_i e^{jk_b x}, \quad (2)$$

where k_b is the wave number of the bending wave on the panel such that $k_b = k \sin(\theta)$ as shown in Figure 1. Hence the specific acoustic impedance of panels one and two can be found from Equations (3) and (4) respectively

$$Z_1 = \frac{2p_i - p}{v_1}, \quad (3)$$

$$Z_2 = \frac{p}{v_2}, \quad (4)$$

where the cavity pressure (p) is created due to the motion of both wall panels. For frequencies below the first resonance frequency perpendicular to the wall panel, Fahy (1985) showed that it

can be assumed that the sound pressure across the depth within the wall cavity is uniform once the depth of the cavity is small compared to the wavelength of the incident waves.

The rate of volume addition per unit volume ($q(x)$) due to the movement of the panels in the y -direction can be found by considering Equations (3) and (4), such that:

$$q(x) = \frac{v_1 - v_2}{d} = \frac{1}{d} \left(\frac{2p_i - p(x)}{Z_1} - \frac{p(x)}{Z_2} \right) \quad (5)$$

where d is the depth of the wall cavity. The situation considered in this paper is governed by the following extension of Gösele and Gösele's (1977) Equation (11),

$$\frac{j\omega\rho_0}{\mathcal{E} + j\omega\rho_0} \frac{d^2 p(x)}{dx^2} + k^2 p(x) = -j\omega\rho_0 q(x) \quad (6)$$

where ($\mathcal{E}$) is flow resistivity of the porous sound absorbing material in the wall cavity and the last term in Gösele and Gösele's (1977) Equation (11) is replaced by the $q(x)$ of this paper. The error in the sign of this last term in Gösele and Gösele's (1977) Equation (11) has been corrected. The above equation extends Gösele and Gösele's (1977) theory by including the limp wall panel's mass impedance and radiation efficiency as well as the effect of pressure doubling and fluid loading. The solution for the wave equation given in Equation (6) showing the effect of the reflected waves within the cavity is given in the next section.

A. Solution to governing equations

Substituting Equation (5) into Equation (6) and gives

$$\frac{d}{\mathcal{E} + j\omega\rho_0} \frac{d^2 p}{dx^2} + \left(\frac{dk^2}{j\omega\rho_0} - \frac{Z_2 + Z_1}{Z_1 Z_2} \right) p(x) = -\frac{2p_i}{Z_1}, \quad (7)$$

with the general solution being in the form

$$p(x) = p_{forced}(x) + p_{reflected}(x).$$

The **reflected** wave solution for the system is given by

$$p_{reflected}(x) = \eta_1 e^{\gamma x} + \eta_2 e^{-\gamma x}, \quad (8)$$

where

$$\gamma = \alpha + j\beta \quad (9)$$

$$\alpha = \frac{1}{\sqrt{2}} \sqrt{\sqrt{a'^2 + b'^2} + a'^2} \quad (10)$$

133 And

$$\beta = \frac{1}{\sqrt{2}} \sqrt{\sqrt{a'^2 + b'^2} - a'^2} \quad (11)$$

134 with

$$a' = \frac{Z_2 + Z_1}{Z_2 Z_1} \frac{\Xi}{d} - k^2 \quad (12)$$

135 and

$$b' = \frac{\Xi k^2}{\omega \rho_0} + \frac{Z_2 + Z_1}{Z_2 Z_1} \frac{\omega \rho_0}{d}. \quad (13)$$

136 Equations (9) to (12) describe the rate of attenuation and the wave-length of the reflected
 137 waves in the cavity. The real part of γ gives the rate of attenuation and the imaginary part gives
 138 the wave number. The value of γ affects the radiation efficiency of the reflected waves and
 139 consequently the sound power radiated.

140 For the **forced** solution, the sound wave incident on panel 1 at an angle θ to the normal can be
 141 written as $\hat{p}_i e^{jk_b x}$, where

$$k_b = k_x = k \sin \theta. \quad (14)$$

142 Equation (7) becomes

$$\frac{d}{\Xi + j\omega \rho_0} \frac{d^2 p(x)}{dx^2} + \left(\frac{dk^2}{j\omega \rho_0} - \frac{Z_2 + Z_1}{Z_1 Z_2} \right) p(x) = -\frac{2p_i e^{jk_b x}}{Z_1}. \quad (15)$$

143 Assuming that $p_f = N e^{jk_b x}$ is the forced solution for the cavity pressure, the forced wave
 144 amplitude (N) can be found by substituting this particular solution into Equation (15) to give

$$N = \frac{j\omega\rho_0 2p_i}{Z_1 d} \left(\frac{1}{\left[k_b^2 \frac{j\omega\rho_0}{\mathcal{E} + j\omega\rho_0} - k^2 + \frac{j\omega\rho_0}{d} \frac{Z_2 + Z_1}{Z_1 Z_2} \right]} \right). \quad (16)$$

As a result the full solution to the wave equation given by Equation (6) can be written in the following form

$$p(x) = N e^{jk_b x} + \eta_1 e^{\gamma x} + \eta_2 e^{-\gamma x}. \quad (17)$$

The coefficients η_1 and η_2 can be found by assuming rigid boundary conditions such that $v(0) = v(l) = 0$; this implies that $dp(x)/dx = 0$ at $x = 0$ and $x = l$.

Therefore;

$$\eta_1 = \frac{jk_b N (e^{-\gamma l} - e^{jk_b l})}{\gamma (e^{\gamma l} - e^{-\gamma l})}, \quad (18)$$

$$\eta_2 = \frac{jk_b N (e^{\gamma l} - e^{jk_b l})}{\gamma (e^{\gamma l} - e^{-\gamma l})}. \quad (19)$$

Although Gösele and Gösele (1977) indicated that the solution for the sound pressure within the wall cavity is in the form shown in Equation (17), they did not give the solution for η_1 and η_2 as given by Equations (18) and (19) respectively. Instead Gösele and Gösele combined the effect of the reflected waves into one equation. Consequently, the solutions presented here for η_1 and η_2 provide an alternative approach to Gösele and Gösele's which will be vital in the study of the effect of the mass-air-mass resonance frequency on the Sound Transmission Loss (STL) of double-leaf wall systems. This effect is crucial in understanding why London's infinite model under-predicts the STL as explained in Section V.

158 III. FORCED RADIATION EFFICIENCY OF FINITE PANELS

The wall panels are modelled as a vibrating strip placed within an infinite baffle as shown in Figure 2. The radiated sound power over angles of radiation (φ) between $\pi/2$ and $-\pi/2$ is required in order to determine the forced radiation efficiency. The radiation efficiency (σ) is,

$$\sigma = \frac{W_{rad}}{\rho_0 c S |v_n|^2} \quad (20)$$

where W_{rad} is the radiated sound power, S is the area of unit length of the strip and v_n is the normal root mean square (rms) velocity.

The radiated sound power can be determined from the radiated sound pressure for an observer at a distance (r) away from the vibrating strip. The distance (r) is much greater than the distance (x) between any two of the infinitesimal width mini-strips which make up the vibrating strip shown in Figure 2; while the distance (R) is the distance from the centre of the strip to an observer in the far field.

Figure 2 Two infinitesimal width mini-strips on a vibrating strip in an infinite baffle (color online)

The **radiated sound pressure** from the strip can be found as follows. For small values of $k\delta x$ (i.e. the wave number of sound in air k multiplied by the width of the infinitesimal width mini-strip, δx) the directivity of a vibrating mini-strip is almost omni-directional. As a result, the radiated sound pressure is approximately equal to the radiated sound pressure of a zero order cylindrical source of the same strength. Using the results of Jacobsen and Juhl, (2011) it can be shown that the sound pressure at a distance (r) away from the surface of the vibrating mini-strip is

$$p(r) = \frac{U_0 \rho_0 c \pi k \delta r}{2} \sqrt{\frac{2}{\pi k r}} e^{j(-kr + \frac{\pi}{4})}. \quad (21)$$

The volume velocity per unit length q_0 of a cylinder of radius δr is equal to $2\pi\delta r U_0$. Since the radiation into the half space is of interest, $q_0 = \pi\delta r U_0$ in this case. Substituting the half space volume velocity into Equation (21) gives

$$p(r) = \frac{q_0 \rho_0 c k}{2} \sqrt{\frac{2}{\pi k r}} e^{j(-kr + \frac{\pi}{4})}. \quad (22)$$

Fahy (1985) stated that it can be assumed that the sound field produced by a small volume velocity source is independent of the detailed form of distribution of velocity over the source surface. For an element δx of the infinitely long narrow strip the volume velocity, $q_0 = v_n \delta x$. Therefore, Equation (22) can be re-arranged to give

$$p(x, r) = \rho_0 c \sqrt{\frac{k}{2\pi r}} e^{j(-kr + \frac{\pi}{4})} v_n \delta x. \quad (23)$$

From Figure 2, if the observer is at a distance $r = R - \frac{l}{2} \sin(\varphi) + x \sin(\varphi)$, Equation (23) can be written as

$$p(x, \varphi) = \rho_0 c \sqrt{\frac{k}{2\pi\left(R + \frac{l}{2}\sin(\varphi) - x\sin(\varphi)\right)}} e^{j\left(-k\left(R + \frac{l}{2}\sin(\varphi) - x\sin(\varphi)\right) + \frac{\pi}{4}\right)} v_n \delta x. \quad (24)$$

188 Assuming that the distance from the centre of the cylinder/strip (R) is significantly greater than

189 $\frac{l}{2}\sin(\varphi) - x\sin(\varphi)$, it can be assumed that $\sqrt{R + \frac{l}{2}\sin(\varphi) - x\sin(\varphi)} = \sqrt{R}$. Consequently,

190 Equation (24) can be written as

$$p(x, \varphi) = \rho_0 c \sqrt{\frac{k}{2\pi R}} e^{j\left(-k\left(R + \frac{l}{2}\sin(\varphi) - x\sin(\varphi)\right) + \frac{\pi}{4}\right)} v_n \delta x. \quad (25)$$

191 The sound pressure radiated from the finite width strip can be found by integrating over the

192 width of the finite width strip such that

$$p(x, \varphi) = \rho_0 c \sqrt{\frac{k}{2\pi R}} \int_0^l e^{j\left(-k\left(R + \frac{l}{2}\sin(\varphi) - x\sin(\varphi)\right) + \frac{\pi}{4}\right)} v_n dx. \quad (26)$$

193 For the forced wave

$$v_n = v_{forced} e^{jk_b x}, \quad (27)$$

194 where v_{forced} is the amplitude of the normal velocity of the forced wave and $k_b = k\sin(\theta)$ is

195 the wave number of the forced wave on the strip. Consequently, the radiated sound pressure due

196 to the forced bending wave can be found by substituting Equation (27) into Equation (26) such

197 that

$$p_{forced}(x, \varphi, \theta) = \rho_0 c \sqrt{\frac{k}{2\pi R}} e^{j\left(-k\left(R + \frac{l}{2}\sin(\varphi)\right) + \frac{\pi}{4}\right)} v_{forced} \int_0^l e^{jx(k\sin(\varphi) + k_b)} dx. \quad (28)$$

198 Solving the integral in Equation (28) gives

$$p_{forced}(\varphi, \theta) = \rho_0 c \sqrt{\frac{k}{2\pi R}} e^{j(-k(R+\frac{l}{2}\sin(\varphi)) + \frac{\pi}{4})} v_{forced} \left[\frac{(e^{jl(k\sin(\varphi) + k_b)} - 1)}{j(k\sin(\varphi) + k_b)} \right]. \quad (29)$$

199 Jacobsen and Juhl (2011) showed that in the far field both the sound pressure and particle
 200 velocity are in phase. As a result, the radiated sound power per unit length over all angles of
 201 radiation is

$$W_{rad} = R \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} I d\varphi = R \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{|p|^2}{\rho c} d\varphi. \quad (30)$$

202 The pressure modulus squared of Equation (29) can be written as

$$|p(\varphi, \theta)|^2 = \rho_0^2 c^2 \frac{k}{\pi R} v_{forced}^2 \frac{1 - \cos(l(k\sin(\varphi) + k_b))}{(k\sin(\varphi) + k_b)^2}. \quad (31)$$

203 Substituting Equation (31) into Equation (30) gives

$$\sigma_{forced}(\varphi, \theta) = \frac{kl}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \text{sinc}^2 \left(\frac{kl}{2} (\sin(\varphi) + \sin(\theta)) \right) d\varphi. \quad (32)$$

204 The forced radiation efficiency (in dB) of the two-dimensional vibrating strip can be seen
 205 in Figure 3. The results shown are similar to that obtained by both Davy (2009a) and Sato (1973)
 206 with the corresponding finite radiation efficiency at grazing incidence (i.e. 90°) being observed.

Figure 3 Forced radiation efficiency of a vibrating strip (color online)

A two-dimensional vibration strip model has been derived and developed for the radiation efficiency of forced waves. The approach presented here is different from that used by Davy (Davy, 2009a), Ljunngren (1991), Sato (1973) and Sewell (1970) in their models for the forced radiation efficiency. During the development of this model it was assumed that:

- The radiated sound pressure from an infinitesimal width vibrating strip is equal to the radiated sound pressure of a zero order cylindrical source with the same volume velocity per unit length
- The radiated sound pressure is independent of the detailed form of distribution of the volume velocity over the surface
- The radiated sound pressure and the particle velocity are in phase

The forced wave which is radiated into the wall cavity will produce reflected waves with different wavelengths and rates of attenuation. As a result, the radiation efficiency of the reflected waves will be different from that of the forced wave. Consequently, in Section IV an iterative numerical approximation is derived for the radiation efficiency of the reflected waves within the wall cavity. An iterative approach is needed because the radiation efficiency of the wave in the wall leaf forced by a reflected wave in the cavity is dependent on its rate of attenuation ($Re(\gamma)$), which is dependent on the impedance of the wall panels, depth and airflow

226 resistivity of the wall cavity (see Equation (9)). The wall panel impedance is dependent on the
 227 fluid loading effect which is determined by the radiation efficiency of the reflected waves.
 228 Therefore, in order to determine the radiation efficiency of these waves, initial approximations
 229 must be made for both γ and their radiation efficiencies.

230 IV. RADIATION EFFICIENCY OF THE REFLECTED CAVITY WAVES

231 The radiated sound pressure in the far field due to the reflected waves in the wall cavity
 232 which excite the wall panels can be solved by substituting the normal velocity due to the
 233 reflected waves into Equation (26). The normal velocity due to the reflected waves is

$$v_{n,\pm} = v_{o\pm} e^{\pm\gamma x} \quad (33)$$

234 where the $\pm$ corresponds to the signs used for γ shown in Equation (17) and $v_{o\pm}$ is the amplitude
 235 of the two reflected waves. Substituting Equation (33) into Equation (26) gives,

$$p_{\gamma\pm}(x, \varphi, t) = \rho c \sqrt{\frac{k}{2\pi R}} e^{j(\omega t - k(R + \frac{l}{2}\sin(\varphi)) + \frac{\pi}{4})} v_{o\pm} \int_0^l e^{jx(k\sin(\varphi))} e^{\pm\gamma x} dx. \quad (34)$$

236 Because the radiation efficiency depends on γ and γ depends on the radiation efficiency, the
 237 correct values are obtained by iteration. As a first approximation, the amplitude and phase of the
 238 reflected waves are assumed initially to be constant along the length of the cavity. This means
 239 that γ is assumed to be zero initially. Utilizing this assumption and following the same procedure
 240 shown in the proceeding section it can be shown that the radiated power and radiation efficiency
 241 of the reflected waves are given by,

$$W_{\gamma=0}(\varphi) = \rho c \frac{k}{\pi l} v_{o\pm}^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1 - \cos(kl\sin(\varphi))}{\sin^2 \varphi} d\varphi. \quad (35)$$

242 and

$$\sigma_{\gamma=0}(\varphi) = \frac{kl}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \text{sinc}^2\left(\frac{kl}{2} \sin(\varphi)\right) d\varphi. \quad (36)$$

Equation (36) gives the first approximation for the radiation efficiency of the reflected waves under the assumption that $\gamma = 0$.

γ can now be determined by substituting this first approximation for the radiation efficiency of the reflected waves into the forced wave impedance of the wall panel. The radiated power and radiation efficiency can then be found from the following

$$W_{\gamma\pm}(\varphi) = \rho c \frac{k}{\pi} v_{o\pm}^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{2 \sin^2\left(\frac{l}{2} (k \sin(\varphi) \mp j\gamma)\right)}{(k \sin(\varphi) \mp j\gamma)^2} \right) d\varphi. \quad (37)$$

and

$$\sigma_{\gamma\pm}(\varphi) = \frac{kl}{2\pi} \frac{v_{o\pm}^2}{|v_{n\pm}^2|} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\text{sinc}^2\left(\frac{l}{2} (k \sin(\varphi) \mp j\gamma)\right) \right) d\varphi. \quad (38)$$

Now since $v_{n\pm} = v_o e^{\pm\gamma x}$ and γ is a complex number, the magnitude of $v_{n\pm}$ can be found from the following

$$|v_{n\pm}| = v_{o\pm} |e^{\pm\gamma x}| = v_{o\pm} |e^{\pm\gamma_{real} x}| |e^{\pm j\gamma_{imaginary} x}| = v_{o\pm} |e^{\pm\gamma_{real} x}| \quad (39)$$

Therefore $|v_{n\pm}|^2$ integrated over the length of the strip can be found from

$$|v_{n\pm}|^2 = v_{o\pm}^2 \int_0^l |e^{\pm 2\gamma_{real} x}| = v_{o\pm}^2 \left(\frac{e^{\pm 2\gamma_{real} l} - 1}{2\gamma_{real}} \right). \quad (40)$$

Substituting Equation (40) into Equation (38) gives the radiation efficiency due to the excitation of the panel due to the reflected waves within the wall cavity as

$$\sigma_{\gamma\pm}(\varphi) = \frac{kl}{2\pi} \left(\frac{e^{\pm 2\gamma_{real} l} - 1}{2\gamma_{real}} \right) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\text{sinc}^2\left(\frac{l}{2} (k \sin(\varphi) \mp i\gamma)\right) \right) d\varphi. \quad (41)$$

254 An additional resistance term $r/\rho_o c$ was added to σ_v . For the empty cavity case, the only
 255 damping within the model is provided by the radiation efficiency. Thus it is necessary to include
 256 some extra damping in the model to take account of the damping in the real system because the
 257 limp panel assumption means that the panel also has no damping. This extra included damping is
 258 mostly needed at the mass-air-mass resonant frequency.

259 V. SOUND TRANSMISSION LOSS OF INFINITE DOUBLE-LEAF WALL SYSTEMS

260 The STL is defined simply as the logarithm of the ratio of the incident sound power upon the
 261 wall to the sound power transmitted through it. Written in terms of the interaction of the incident
 262 (p_i) and transmitted sound pressure (p_t) the sound transmission coefficient (τ) for an infinite
 263 panel can be found from

$$\tau = \left| \frac{p_t}{p_i} \right|^2. \quad (42)$$

264 By assuming continuity of the particle velocity, London showed the relationship between the
 265 transmitted pressure of an infinite wall system ($p_{\infty,trans}$) and particle velocity is

$$p_{\infty,trans} = \left| \frac{\rho_o c v_2}{\cos(\theta)} \right|, \quad (43)$$

266 where v_2 is the particle velocity on the panel 2. v_2 is dependent on the pressure within the wall
 267 cavity and the specific acoustic impedance as shown in Equation (4). As a result

$$p_{\infty,trans} = \frac{\rho_o c}{\cos(\theta)} \left| \frac{p_{\infty,cavity}}{Z_2} \right|, \quad (44)$$

268 where

$$p_{\infty,cavity} = |N e^{jk_b x}| = |N|. \quad (45)$$

269 and N is the amplitude of the forced wave given by Equation (16).

270 Beranek (1971) showed that the forced radiation efficiency of an infinite plate (σ_{∞}) due to
 271 airborne excitation is given by

$$\sigma_{\infty} = \frac{1}{\sqrt{1 - \sin^2(\theta)}} = \frac{1}{\cos(\theta)}. \quad (46)$$

272 Consequently Equation (43) can be written in terms of σ_∞ as

$$p_{\infty,trans} = \rho_o c \sigma_\infty \left| \frac{p_{\infty,cavity}}{Z_2} \right|. \quad (47)$$

273 From Equation (47) it can be seen that the transmitted pressure squared is dependent only on the
274 amplitude of the forced waves in the infinite model. This is due to the underlying assumption
275 within this model that the panels and by extension the wall cavity are infinitely long such that
276 waves excited within the system keep travelling without any reflections. Consequently from
277 Equation (17) the sound pressure within the cavity for the infinite model is only dependent on the
278 forced waves as shown in Equation (45).

279 The STL for the infinite model (STL_∞) can be found by assuming that the incident
280 pressure in Equation (42) is equal to one. This assumption can be made as the transmitted
281 pressure is also dependent on the incident pressure; since the STL involves the ratio of these two
282 sound pressures any value can be assumed for the incident sound pressure. As a result, the sound
283 transmission loss for the infinite model can be found from

$$STL_\infty = 10 \log_{10} \left(\frac{1}{p_{\infty,trans}^2} \right). \quad (48)$$

284 The STL prediction obtained from Equation (48) compared to London's model for an empty
285 double-leaf infinite wall system with equal mass of 12.3 kg/m² for both wall panels and a cavity
286 depth of 90 mm can be seen in Figure 4.

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Figure 4 Comparison between the current theory and London's model for an infinite double-leaf wall at different angles of incidence (color online)

The results in Figure 4 show that the predictions produced from the developed model compare well with the results obtained from London's work. The discrepancies between both models at the higher frequencies are due to the effects of the first cross cavity mode which is taken into account in London's model but not in the presented model since a constant sound pressure across the cavity is assumed. This figure also shows that the mass-air-mass resonance frequency varies with angle of incidence, with the lowest of these occurring at the normal mass-air-mass resonance frequency.

VI. SOUND TRANSMISSION LOSS OF FINITE DOUBLE-LEAF WALL SYSTEMS

In Section V it was shown that a model predicting similar results as London's for infinite cavity walls can be obtained by considering only the magnitude of the forced wave in the cavity. However, for the finite wall system both the forced and reflected waves in the cavity dictate the response of the entire system. In this section it is assumed that the radiation efficiency of the vibration created in the leaves of the wall by the reflected waves in the cavity are the same as the radiation efficiency of the vibration created in the leaves of the wall by the forced waves in the cavity. This avoids the need for iteration in the calculations.

A. Incident Sound Power

One major problem associated with the infinite model, is that it predicts that no sound will be transmitted at grazing incidence due to the radiation efficiency of the infinite panels. As a result, some researchers utilize a finite radiation efficiency as a substitute for $1/\cos(\theta)$ when modelling finite wall systems. For example, the wave impedance of an infinite wall panel is equal to the ratio of the incident sound pressure and the normal particle velocity such that

$$Z_i = \frac{p_i}{u_i \cos(\theta)} = \frac{Z_c}{\cos(\theta)} = \frac{\rho c}{\cos(\theta)}, \quad (49)$$

where Z_c is the characteristic impedance of air such that $Z_c = p_i/u_i$. Rindel (1975) showed that the sound power incident on an infinite panel can be found from

$$W_{i\infty} = \int_S \frac{1}{2} \text{Re}(P_i u_{iy}^*) dS = \frac{p_{i,rms}^2 S}{\text{Re}(Z_i)} = \frac{p_{i,rms}^2}{\rho c} S \cos(\theta), \quad (50)$$

where the subscript y corresponds to the co-ordinate system shown in Figure 1, the amplitude variables $P_i u_{iy}^*$ are peak amplitudes rather than rms amplitudes and the area (S) of a unit length of the strip is l . The incident sound power is

$$W_{i\infty} = \frac{p_{i,rms}^2}{\rho c} l \cos(\theta). \quad (51)$$

In order to account for the finite size of the wall panel, the $\cos(\theta)$ term in Equation (51) can be replaced by the inverse of the forced radiation efficiency to give

$$W_i = \frac{p_{i,rms}^2 l}{\rho c \sigma_{forced}}. \quad (52)$$

Rindel (1975) utilized such a substitution and commented that this was similar to the way in which it was employed by Heckl (1964). Rindel stated that the use of the radiation efficiency in this manner also implies that the incident power per unit area is small when kl is large and increases when kl decreases. The deformation of the sound field by diffraction effects was given as the explanation for the reason why this occurs. The use of the finite radiation efficiency in this way is similar to spatial windowing technique developed by Villot et al. (2001). In this initial introduction of the theory, the spatial window was applied to both the sound pressure field and the vibration before calculating the radiated field. According to Vigran (2009) and Allard and Atalla (2009) this technique was modified by Villot and Guigou-Carter (2005) by only taking the spatial window into account on the transmitted side; an approach which Vigran (2009) employed

when developing his one dimensional finite radiation impedance method. Ghinet and Atalla (2001) also created an alternative method to the Villot et al. (2001) spatial windowing technique by using a finite rather than an infinite forced radiation efficiency to correct the infinite sound transmission coefficient to a finite one.

However, despite Rindel's (1975) and Villot *et al*'s (2001) successful substitution of the finite radiation efficiency in the manner described, this approximation cannot be done on both the transmitted power and the incident power. This was the reason for Villot and Guigou-Carter's (2005) correction. Furthermore, if this approximation is done for the incident power, it should only be utilized when finding the angular dependent sound transmission loss and not when finding the diffuse sound field sound transmission loss. This is because the $\cos(\theta)$ term used when calculating the diffuse sound field transmission coefficient (as shown in Equation (60)) represents the projected area of the sound field onto the wall panel. Consequently, a further substitution of the finite radiation efficiency for this $\cos(\theta)$ term cannot be done. As a result, Equation (51) should be used when predicting the diffuse field STL while Equation (52) should be utilized when calculating the angular dependent STL. The use of the radiation efficiency in this manner has been validated by Brunskog (2012) while evaluating Davy's (2009b) theory.

B. Total Transmitted Sound Power

The total transmitted sound power can be found from the radiated sound pressure which is dependent on both the forced and reflected waves within the cavity. Consequently, the total radiated sound pressure of the vibrating strip can be found by replacing the velocity in Equation (28) with the total velocity due to both the forced and reflected waves such that

$$p(x, \varphi, \theta, t) = \rho c \sqrt{\frac{k}{2\pi R}} \int_0^l e^{j(\omega t - k(R + \frac{l}{2} \sin(\varphi) - x \sin(\varphi)) + \frac{\pi}{4})} v_{n,total}(x) dx, \quad (53)$$

352 where,

$$v_{n,total}(x, \theta) = v_{forced} e^{jk_b x} + v_{o+} e^{\gamma x} + v_{o-} e^{-\gamma x}. \quad (54)$$

353 Furthermore, it can be shown that the magnitude of the pressure squared is given by

$$|p(\varphi, \theta)|^2 = \rho^2 c^2 l \left(\frac{k}{2\pi R} \right) \left| v_{forced} \text{sinc}\left(\frac{A}{2}\right) + v_{o+} \text{sinc}\left(\frac{B}{2}\right) + v_{o-} \text{sinc}\left(\frac{C}{2}\right) \right|^2 \quad (55)$$

354 where,

$$A = kl \sin(\varphi) + k_b,$$

$$B = kl \sin(\varphi) - j\gamma l,$$

and

$$C = kl \sin(\varphi) + j\gamma l.$$

355 Consequently,

$$W_{total}(\varphi, \theta) = \rho c l \left(\frac{k}{2\pi} \right) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left| v_{forced} \text{sinc}\left(\frac{A}{2}\right) + v_{o+} \text{sinc}\left(\frac{B}{2}\right) + v_{o-} \text{sinc}\left(\frac{C}{2}\right) \right|^2 d\varphi. \quad (56)$$

356 The velocity amplitude in Equation (56) for the forced and reflected waves can be found from

$$v_{forced} = \frac{N}{Z_2}, \quad (57)$$

$$v_{o+} = \frac{\eta_1}{Z_2}, \quad (58)$$

$$v_{o-} = \frac{\eta_2}{Z_2}, \quad (59)$$

357 where N is the pressure amplitude due to the forced wave (Equation (16)), η_1 and η_2 are the

358 pressure amplitudes due to the reflected waves as given in Equations (18) and (19) respectively:

Z_2 is the impedance of the Panel 2 with the appropriate radiation efficiency used for the fluid loading effect.

C. Physical Explanation For The Under-Prediction Of London's Model

Based on the above discussion the angular dependent transmission coefficient can be found from the ratio of the total transmitted power given by Equation (56) to the incident power given by Equation (52). The resulting angular dependent STL results for a 3.05 by 2.44 m gypsum double wall system with a 90 mm cavity can be seen in Figure 5. A cavity length of 2.44 m was utilized when calculating the results shown.

Figure 5 Comparison between London's infinite model and the proposed finite model assuming that the forced and reflected waves have the same radiation efficiency (color online)

The results in Figure 5 show that the oblique mass-air-mass resonance frequency for the finite model does not vary with the angle of incidence as in London's model. Instead, for all angles of incidence the oblique mass-air-mass resonance frequency occurs at the normal mass-air-mass frequency. The presence of the reflected waves within the wall cavity is responsible for the normal mass-air-mass resonance frequency being independent of the angle of incidence. The dips in the STL above the normal mass-air-mass resonance frequency shown in Figure 5 are due

to the cavity resonances associated with the finite size of the wall cavity. The results shown indicate that these resonance frequencies do not vary with the angle of incidence.

In order to determine how the prediction results compare to measurements reported by Warnock (2010) the diffuse sound field sound transmission coefficient (τ_d) was calculated using Equation (60) and averaged into 1/3 octave bands.

$$\tau_d = 2 \int_0^{\pi/2} \tau \sin(\theta) \cos(\theta) d\theta \quad (60)$$

The results for a 16 mm gypsum (density 770 kg/m³) double-leaf walls with a 90 mm cavity without sound absorbing material in the wall cavity can be seen in Figure 6. The measured sound insulation results TL-92-265 to TL-92-267 (from Warnock(2010)) in Figure 6 contain 40 mm double steel studs spaced at 610 mm centers with a 10 mm gap between the separate studs, while TL-92-262 to TL-92-264 contain 90 mm steel studs on 813 mm centers. These walls have critical frequencies of 2.5 kHz.

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Figure 6 Measured and calculated sound transmission loss in 1/3 octave bands (color

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online)

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The results in Figure 6 clearly show the extent by which London's model underestimates the STL through double-leaf wall systems due to the angular dependent mass-air-mass resonance. Once this effect is removed by taking into account the interaction of the reflected waves within the wall cavity due to its finite size, more realistic predictions are obtained. Therefore, the physical explanation for the reason why London's model under-predicts the STL comes from the effect of the angular dependent mass-air-mass resonance frequency.

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The link between the angular dependent mass-air-mass resonance frequency and London's under-prediction of the STL has not been reported within the literature. Although

Prasetiyo and Thompson's (2012) recent publication which is based on the coupled Waveguide Finite Element-Wavedomain Boundary Element method to predict the STL showed that the oblique resonance does not vary with the angle of incidence (an observation which agrees with the results presented here); they could not use their results to explain the reason why London's model under-predicted the STL since they were not able to predict the normal mass-air-mass resonance frequency with their model. Instead, their first resonance occurred at the point where the first cavity resonance (i.e. $n_x = 1$) combined with the normal mass-air-mass resonance frequency. Despite this fact, even though the technique presented here is different from Prasetiyo and Thompson's (2012), the results obtained from both models indicate that the lateral cavity modes do have a significant impact on the prediction results obtained for the STL through double-leaf wall systems.

VII. SOUND TRANSMISSION LOSS OF AN EMPTY DOUBLE-LEAF WALL

In the previous section it was assumed that the same radiation efficiency could be used for the vibrations created by both the forced and reflected forced waves. This assumption is not entirely valid as the radiation efficiencies of both waves are different since their wavelengths and rates of attenuation are not the same. The predicted STL when considering the radiation efficiency of the reflected waves can be seen in Figure 7. The effect of considering the different radiation efficiencies of the reflected waves can be seen by comparing the predicted STL shown in Figure 6 and Figure 7.

Figure 7 STL of a 16 mm gypsum double-leaf wall with a 90 mm cavity without sound absorption material (color online)

The results shown in Figure 7 show the predicted results with air flow resistivity of 0 and 50 Ns/m^4 . For the presented model an air-flow resistivity of 50 Ns/m^4 is necessary because the empty cavity provides some absorption of sound in addition to that provide by the radiation of sound from the outer sides of each wall panel. This result is consistent with Gösele and Gösele's (1977) result that the radiation of sound from one wall is equivalent to an airflow resistivity of between 1 and 10 Ns/m^4 .

In order to obtain the results shown in Figure 7, a resistance term of $r = 600 \text{ kg}/(\text{m}^2 \text{ s})$ for the vibration of the wall leaves was included in the model to reduce the extent of the singularity obtained at the mass-air-mass resonance frequency (f_0). The effect of different values of this resistance term r on the STL can be seen in Figure 8. This resistance term is needed physically because real wall panels will have some damping.

The error due to the iterative calculation of the radiation efficiency of the reflected waves, is expressed as the approximate percent relative error (ε_a) which is given by

$$\varepsilon_a = \frac{\text{present approximation} - \text{previous approximation}}{\text{present approximation}} 100\%, \quad (61)$$

Chapra and Canale (2002), for successive iterations around f_o is given in Table 1. From the results shown in Table 1 it can be seen that with successive iterations the diffuse sound field STL rapidly converges. Similar rates of convergence were obtained for the STL at discrete frequencies for different angles of incidence as well as for the radiation efficiency of the reflected waves. As a result of the small ε_a which occurs by the 4th iteration, only four iterations were used for all the prediction results presented in this paper.

Figure 8 Effect of the resistance term r ($\text{kg}/(\text{m}^2 \text{ s})$) on the predicted STL for a 16 mm double-leaf gypsum wall system without sound absorption material (i.e. airflow resistivity= $50 \text{ N s}/\text{m}^4$) (color online)

Table 1 Approximate percent relative error of the STL for successive iterations of the radiation efficiency of the reflected waves

1/3 Octave Band Centre Frequency (Hz)	Iteration 2	Iteration 3	Iteration 4
80	2.62E-02	-4.56E-04	8.28E-06
100	-1.21E-02	6.52E-05	-3.49E-07
125	-2.02E-03	2.69E-06	1.80E-09
160	5.63E-04	-1.03E-06	1.91E-09
200	-1.21E-03	1.57E-06	-2.05E-09

The results in Figure 7 show that the derived model accurately predicts the STL for the empty double-leaf wall system below and above f_0 up until approximately half of the critical frequency of the wall panel once suitable resistivity (i.e. Ξ) and resistance (i.e. r) is included.

VIII. SOUND TRANSMISSION LOSS OF A FULLY FILLED DOUBLE-LEAF WALL

Research conducted by Gösele and Gösele (1977), Novak (1992), Narang (1993; 1995) and Royar (2007) showed that an airflow resistivity of approximately 5000 Ns/m⁴ is needed to damp the modes within the wall cavity. Above this airflow resistivity little improvement in the STL is obtained. The developed model was used to verify this trend for a 16 mm double-leaf wall gypsum board system with varying airflow resistivity within a 90 mm cavity as shown in Figure 9. These results indicate that as the airflow resistivity is increased, the STL steadily increases up until an airflow resistivity of approximately 5000 Ns/m⁴. Above this airflow resistivity little improvement in the STL occurs; a result which is supported by other work within the literature. However, although the developed model followed the expected trend, in order to determine the accuracy of the model, comparisons were made to experimental data.

Figure 9 Effect of different airflow resistivity (color online)

The prediction for the STL of a 16 mm gypsum double-leaf wall system, with glass-fibre (flow resistivity 4800 Ns/m^4) within a 90 mm cavity, can be seen in Figure 10. TL-92-268 to TL-92-274 contain 40 mm double steel studs spaced at 610 mm centers with a 10 mm gap between the separate studs, while TL-92-275 does not contain any studs, Warnock (2010). These walls have a critical frequency of 2.5 kHz.

Figure 10 STL of a double-leaf wall system with glass-fibre within the 90 mm wall cavity (color online)

Figure 10 shows the predicted results with the measured glass-fibre air flow resistivity of 4800 Ns/m⁴ as well as the results obtained when 20% of the measured airflow resistivity of the material was utilized. The reduction in the glass-fibre airflow resistivity was necessary because a higher than expected STL was obtained when the actual airflow resistivity of 4800 Ns/m⁴ was used in the developed model as shown in Figure 10. A similar problem was observed by Novak (1992) within his transfer matrix model which utilized the airflow resistivity of the material in the wall cavity in order to predict the sound insulation of a cavity wall. Novak did not give an explanation for this result. However, with regard to the proposed model there are three plausible reasons why the overestimate occurs when the actual flow resistivity is used.

1. The reported airflow resistivity is usually measured normal to the surface of the material and not in a planar direction. Allard and Atalla (2009) noted that fibrous materials are

generally anisotropic and the fibres generally lie in planes parallel to the surface of the material. As a result, the normal airflow resistivity ($\mathcal{E}_n$) which is measured perpendicular to the planes of the fibres is different from the planar airflow resistivity ($\mathcal{E}_p$) which is measured parallel to the directions of the planes. Allard (1987) discussed the work of Burke (1983) and Nicholas and Berry (1984) which showed that the ratio of the planar to normal airflow resistivity was approximately 0.5. Consequently, since the developed model was derived by integrating along the length of the cavity rather than across its depth, the planar airflow resistivity should be utilized within the model rather than the usual reported normal airflow resistivity. This issue relating to direction of the measured airflow resistivity gives a plausible explanation for the reason why at most 50% of the measured normal airflow resistivity should be used within the developed model. This issue has also been studied by Tran-Van (2004).

2. The assumption within the developed model that the sound absorbing material within the wall cavity does not move when excited by the sound waves. Schultz (Beranek, 1971) discussed the properties of porous materials when excited by sound waves. In this discussion Schultz noted that in the low frequency region if the material has insufficient inertia to remain motionless it will move as a whole because of the action of the air particles pumped back and forth by the sound pressure through the pores of the blanket. Schultz concluded that under such circumstances the blanket can be treated in terms of lumped constants with no consideration of sound propagation within the blanket (Beranek, 1971). No sound propagation within the blanket means that the sound absorption properties will be drastically reduced. Consequently, if there is any movement of the sound absorption material within the wall cavity for the frequency range

considered, the airflow resistivity required for these calculations would be significantly lower. Although it can be argued that the movement of the sound absorption material may not significantly affect the predictions from the developed model since the movement of the material in the normal direction will be greater than the planar direction, it must be noted that any movement of the material in any direction will cause some reduction in its sound absorption properties and reduce the required airflow resistivity needed for the model. The extent of this required reduction in airflow resistivity if the sound absorption material moves within the wall cavity is unknown. It may be possible in the future to incorporate the inertia of the sound absorbing material into the model described in this paper using the limp frame equivalent fluid model found in Allard and Atalla (2009).

3. The sound radiation into the wall cavity. The developed model is based on the assumption that below the critical frequency the forced bending waves are efficient radiators while the free bending waves are inefficient. The hydrodynamic short circuiting of the free bending waves within the finite wall panel is responsible for the inefficiency of these waves. However, although this may be true for radiation into a free space or into an empty wall cavity, Tomlinson *et al.* (2004) showed that the radiation efficiency of the free bending waves on a plate increases as the airflow resistivity of the porous medium it is radiating into increases. For example, Tomlinson *et al.* calculated the radiation efficiency of a plate at 100 Hz radiating into a 100 Ns/m⁴ porous medium as 0.018 and approximately 0.3 into a 5000 Ns/m⁴ medium. Clearly the significance of the free bending waves increases as the airflow resistivity increases below the critical frequency. Because the wall leaves are assumed to be limp in this paper, there cannot be reflected

freely propagating bending waves in the wall leaves and these non-existent freely propagating bending waves cannot radiate sound into the wall cavity. If these freely propagating bending waves were allowed to exist, the predicted STL would be less than that given by the current model. It should be noted that none of the models within the literature account for the increase in the radiation efficiency of the free bending waves due to the difference in sound radiation into a porous medium as outlined by Tomlinson *et al.* (2004).

Despite the issues discussed relating to the modelling of the sound absorbing material within the cavity, the results shown in Figure 10 show that once 20% of the airflow resistivity is used for the 90 mm cavity, the developed model accurately predicts the STL above and below f_0 even though at f_0 the results obtained from the prediction does not account for the decrease of f_0 due to the change in the speed of sound through the porous material as discussed by Narang (1993). For wall systems with a 205 mm deep cavity using 40 % of the reported airflow resistivity when more than 50 % of the cavity is filled produces satisfactory results Cambridge (2012) .

SUMMARY AND CONCLUSIONS

A model to determine the STL through both the infinite and the finite double-leaf wall systems by studying the propagation of the forced and reflected waves in the wall cavity has been developed. The infinite model's prediction compared well with London's model, while the deteriorating effect of the angular dependent mass air mass resonance frequency was given as the main reason why London's model under-predicts the STL. The predictions obtained when the difference between the radiation efficiency of the forced and reflected waves was taken into account also compared well to experimental data for both the empty and full cavity case although

an additional resistance term and a reduced airflow resistivity had to be incorporated within the model. The inclusion of the additional resistance term r was needed to reduce the extent of the reduction in the prediction of the STL at the mass air mass resonance frequency. The reduction in the required airflow resistivity was needed because:

- The planar airflow resistivity is approximately 50% of the usually normal airflow resistivity
- There is a possibility that the sound absorption material may move within the cavity, therefore reducing its sound absorption properties
- The radiation of sound by the free bending waves in the wall leaves was not included in the model because of the limp wall leaf assumption due to the belief that they are inefficient radiators below the critical frequency. However, Tomlinson's (2004) investigation suggests that the radiation of sound into the wall cavity by these waves actually increases when the resistivity of the porous material in the wall cavity increases.

The issues encountered while using the airflow resistivity of the material to model its sound absorption characteristics is consistent with that encountered by Novak (1992), although no explanation was given by him for the reason why an over prediction of the STL occurred once the actual normal airflow resistivity was used.

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